Two-Phase Gas/Liquid Pipe Flow

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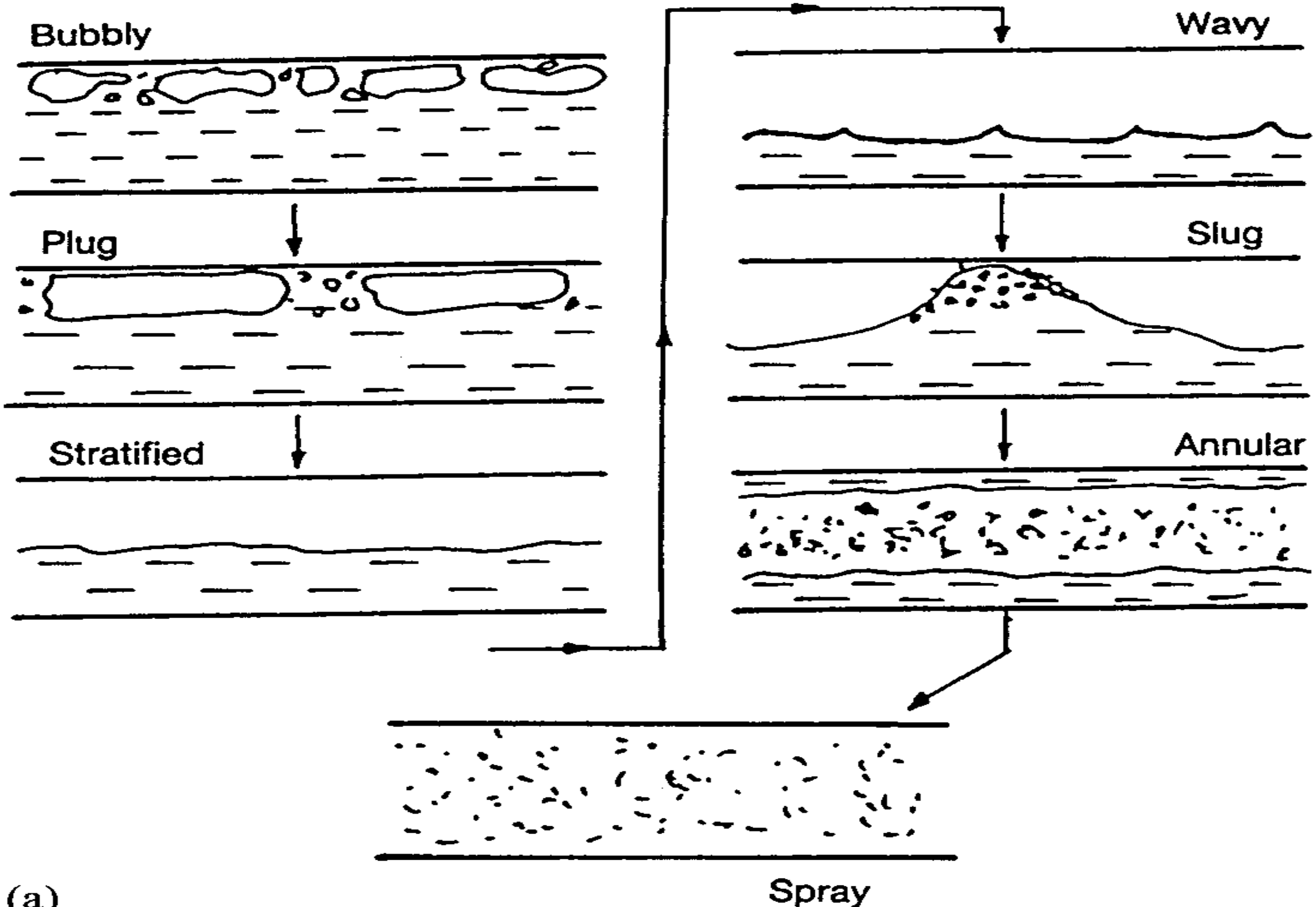
Types of Two-Phase Flow

- ***Solid-Gas***
- ***Solid-Liquid***
- ***Gas-Liquid***
- ***Liquid-Liquid***

Gas-Liquid Flow Regimes

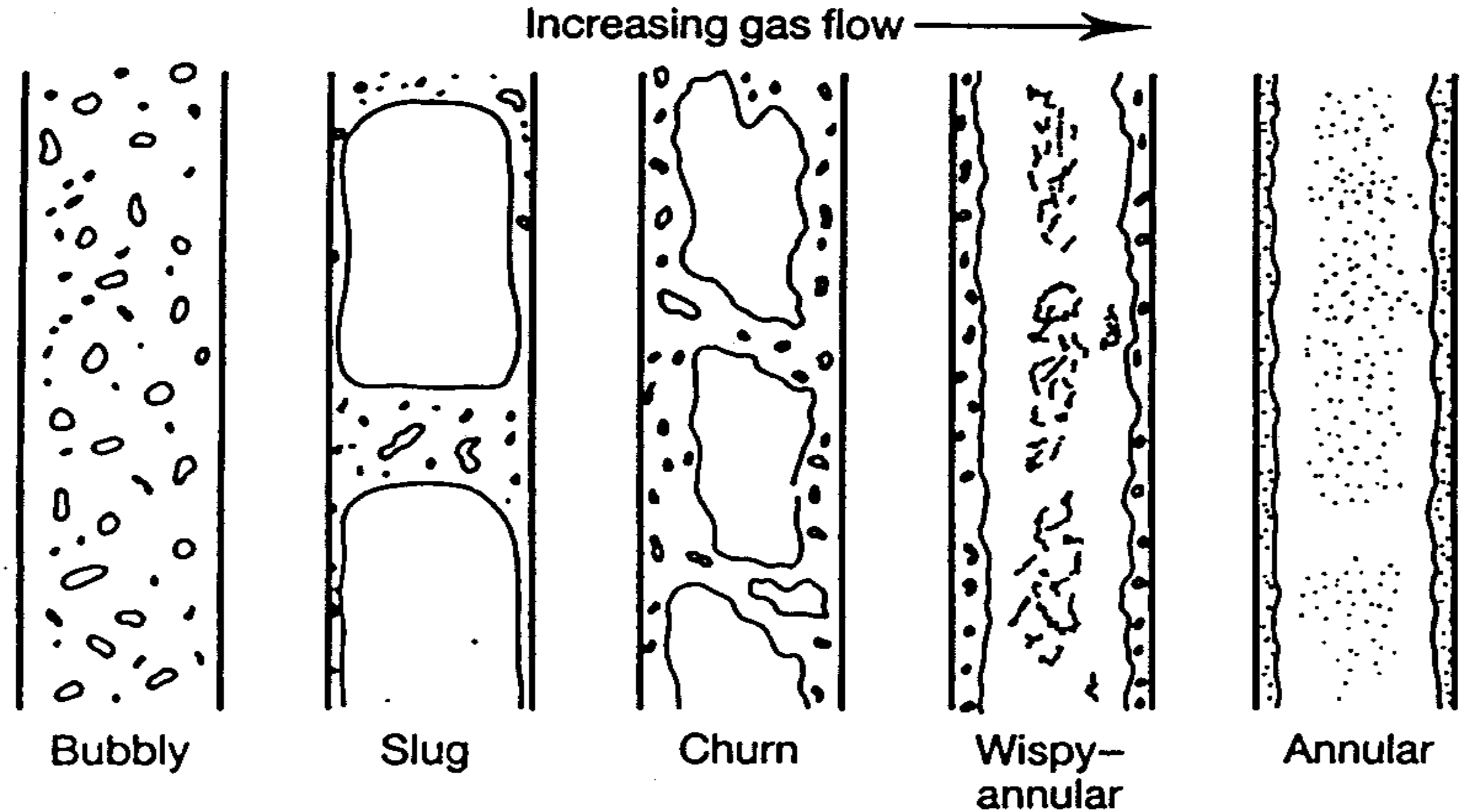
- ***Homogeneous***
  - ***Highly Mixed***
  - ***“Pseudo Single-Phase”***
  - ***High Reynolds Number***

- ***Dispersed –Many Possibilities***
  - ***Horizontal Pipe Flow***
  - ***Vertical Pipe Flow***



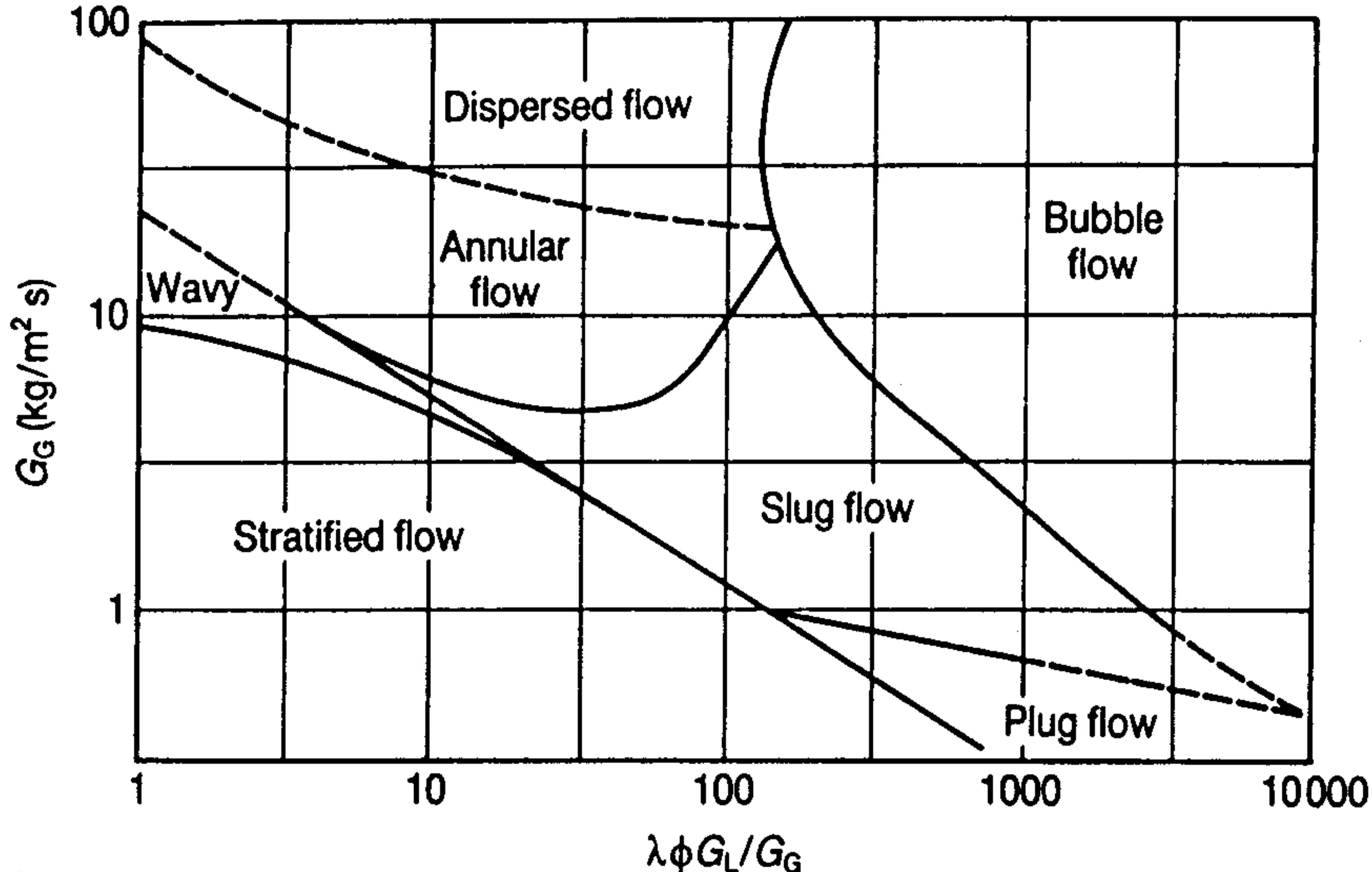
(a)

Horizontal Dispersed Flow Regimes



(b)

Vertical Pipe Flow Regimes

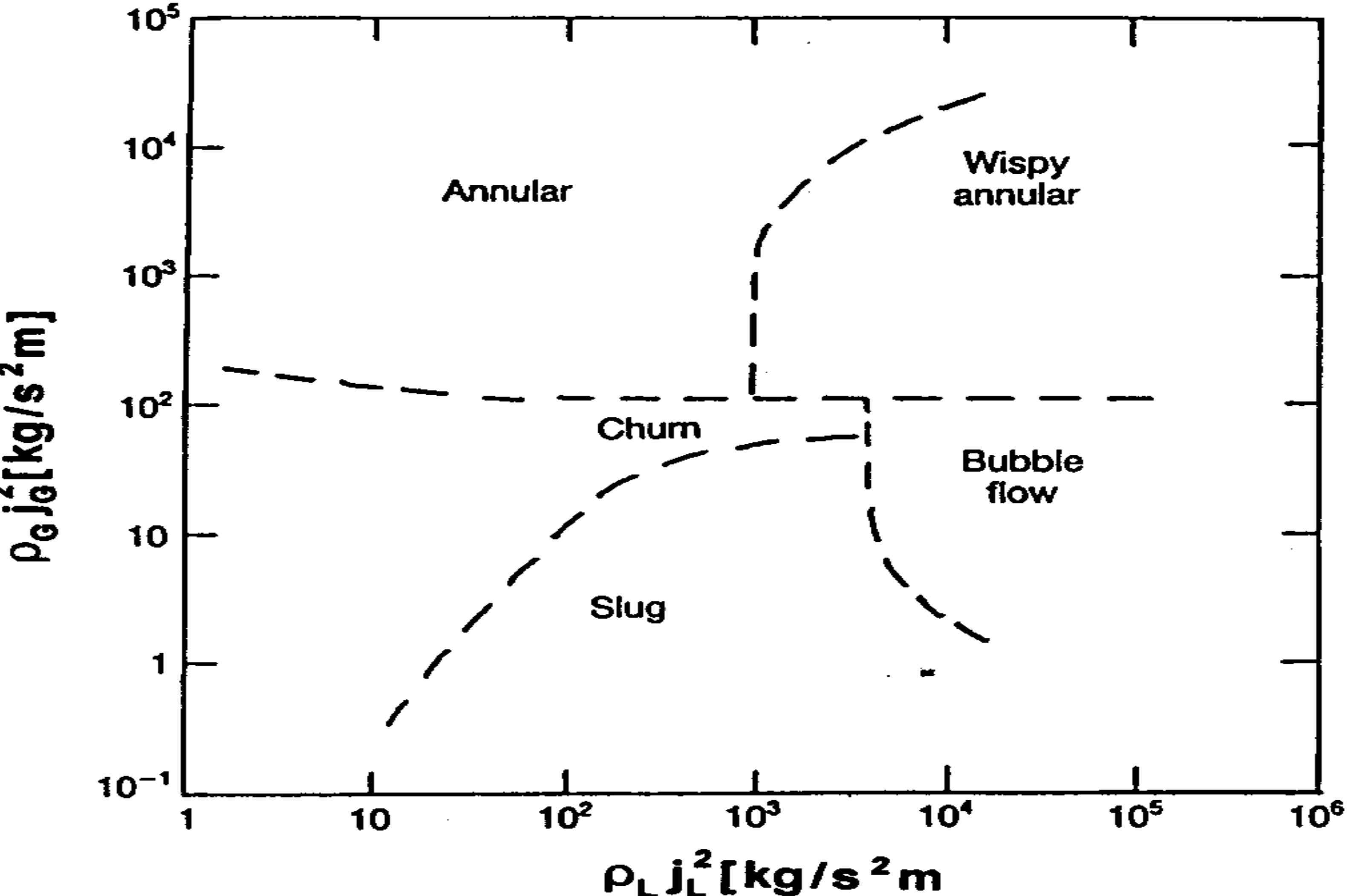


(a)

Horizontal Pipe Flow Regime Map

$$\lambda = \left[\left(\frac{\rho_G}{\rho_A}\right)\left(\frac{\rho_L}{\rho_W}\right)\right]^{1/2}$$

$$\phi = \frac{\sigma_W}{\sigma_L}\left[\frac{\mu_L}{\mu_W}\left(\frac{\rho_W}{\rho_L}\right)^2\right]^{1/2}$$



(b)

Vertical Pipe Flow Regime Map

DEFINITIONS

Mass Flow Rate $(\dot{m})$, Volume Flow Rate (Q)

$$\dot{m} = \dot{m}_L + \dot{m}_G = \rho_L Q_L + \rho_G Q_G$$

Mass Flux (G):

$$G = \frac{\dot{m}}{A} = \frac{\dot{m}_L}{A} + \frac{\dot{m}_G}{A} = G_L + G_G$$

Volume Flux (J):

$$J = J_L + J_G = \frac{G}{\rho_m} = \frac{G_L}{\rho_L} + \frac{G_G}{\rho_G}$$

$$= \frac{Q_L + Q_G}{A} = V_m$$

Volume Fraction Gas: ε

Vol. Fraction Liquid: $1\text{-}\varepsilon$

Phase Velocity:

$$V_L = \frac{J_L}{1-\varepsilon} \quad , \quad V_G = \frac{J_G}{\varepsilon}$$

Slip Ratio (S):

$$S = \frac{V_G}{V_L}$$

Mass Fraction Gas (Quality x):

$$x = \frac{\dot{m}_G}{\dot{m}_G + \dot{m}_L}$$

Mass Flow Ratio Gas/Liquid:

$$\frac{\dot{m}_G}{\dot{m}_L} = \frac{x}{1 - x} = S\left(\frac{\rho_G}{\rho_L}\right)\left(\frac{\varepsilon}{1 - \varepsilon}\right)$$

Density of Two-Phase Mixture:

$$\rho_m = \varepsilon\rho_G + \left(1-\varepsilon\right)\rho_L$$

where

$$\varepsilon = \frac{x}{x + S\left(1-x\right)\rho_G / \rho_L}$$

is the volume fraction of gas in the mixture

Holdup (Volume Fraction Liquid):

$$\varphi = 1 - \varepsilon = \frac{S\left(1-x\right)\left(\rho_G / \rho_L\right)}{x + S\left(1-x\right)\left(\rho_G / \rho_L\right)}$$

Slip (S)

- ***Occurs because the gas expands and speeds up relative to the liquid.***
- ***It depends upon fluid properties and flow conditions.***
- ***There are many "models" for slip (or holdup) in the literature.***

<u>Hughmark</u> (1962) slip correlation

Either horizontal or vertical flow:

$$S = \frac{1 - K + \left[\left(1 - x\right)/x\right]\rho_G/\rho_L}{K\left[\left(1 - x\right)/x\right]\rho_G/\rho_L}$$

where

$$K = \left(1 + 0.12/Z^{0.95}\right)^{-19}$$

$$Z = N_{Re}^{1/6} N_{Fr}^{1/8} / \left(1 - \varepsilon\right)^{1/4}$$

HOMOGENEOUS GAS-LIQUID PIPE FLOW

Energy Balance (Eng'q Bernoulli Eqn)

$$-\frac{dP}{dL} = \frac{\dfrac{2 f_m G^2}{\rho_m D} + G^2 v_{GL}\dfrac{dx}{dL} + \rho_m g\dfrac{dz}{dL}}{1 + xG^2\dfrac{dv_G}{dP}}$$

where

$$\nu_{GL} = \nu_G - \nu_L = 1/\rho_G - 1/\rho_L$$

For "frozen" flow (no phase change):

$$\frac{dx}{dL} = 0$$

If $xG^2 \frac{dv_G}{dP} = -1$ flow is choked.

For ideal gas:

$$\left(\frac{\partial v_G}{\partial P}\right)_T = -\frac{1}{\rho P} \quad , \quad \left(\frac{\partial v_G}{\partial P}\right)_s = -\frac{P_1^{1/k}}{\rho_1 k P^{(1+k)/k}}$$

For frozen ideal gas/liquid choked flow:

$$G = c\rho_m = \sqrt{\frac{\rho_m k P}{\varepsilon}}$$

For flashing flow (Clausius-Clapeyron eqn):

$$\left(\frac{\partial v_G}{\partial P}\right)_T \cong -\frac{v_{GL}^2 c_p T}{\lambda_{GL}^2}$$

Homogeneous Horizontal Flow

$$-dP = \frac{G^2\left(\frac{2f_m}{\rho_m D}dL + v_{GL}dx\right) + \rho_m dz}{\left(1 + xG^2 dv_G / dP\right)}$$

Flashing Flow - Determine x from (adiabatic) energy balance (or thermo properties database):

$$x = \frac{c_p\left(T_o - T_s\right)}{\lambda_{GL}}$$

***Finite Difference Solution** – solve for* ΔL

$$\Delta L = -\frac{D}{4 f_m}\left[\left(\Delta P + G^2 \Delta v + g\Delta Z / v\right)\frac{2}{vG^2} + \Sigma K_{fit}\right]$$

Dimensionless

$$\frac{4\,f_m \Delta L}{D} + \Sigma K_{fit} = -\left[\left(\Delta\eta + G^{*2}\Delta\varepsilon + g\Delta Z / P_o v_o \varepsilon\right)\frac{2}{\varepsilon G^{*2}}\right]$$

where

$$\Delta\eta = \Delta P / P_o$$

$$G^* = G / \sqrt{P_o \rho_o} = G\sqrt{v_o / P_o}$$

$$\varepsilon = v / v_o = \rho_o / \rho$$

$$\left(L = \sum_i (\Delta L)_i\right)$$

Using $\Delta Z = \Delta L \cos\theta$ *, where* θ *is the pipe inclination angle with the vertical*

for horizontal $\cos\theta = 0$

for vertical up flow $\cos\theta = 1$

for vertical down flow $\cos\theta = -1$

$$\frac{4 f_m \Delta L}{D} = -\frac{\left[2\left(\Delta\eta + G^{*2}\Delta\varepsilon\right)/\,\varepsilon G^{*2} + \Sigma K_{fit}\right]}{1 + gD\cos\theta\,/\,4 f P_o v_o \varepsilon}$$

Procedure: *Find G, Given L, P_o and P_e*

- ***Select desired ΔP and determine v at each pressure step from P_o to P_e***
- ***Assume a value for G^****
- ***Calculate ΔL at each pressure step.***
  At choke point, $\Delta L \rightarrow 0$
- ***Adjust until $\Sigma \Delta L = L$***

Ex: Flashing Water in Pipe

Given: $G, P_o, T_o, \left(x_o, s_o . \rho_{Go}, \rho_{Lo}\right)$

Calculate: Pressure Drop over L

1. Assume a value for $dL = \Delta L;$
2. Est. $f_m = fn\left(DG / \mu_m\right)$ (Moody, Churchill, ~0.005)
3. $\Delta P_1 \approx -G^2\left[2 f_m \Delta L / \rho_m D\right]$, $\rho_m = fn\left(\rho_L, \rho_G, x, S\right)$
4. $P_1 = P_o + \Delta P_1 \rightarrow \left(x_1, T_{s1}, \rho_{G1}, \rho_{L1}\right)_s \rightarrow \Delta x_1, v_{GL_1} = \frac{1}{\rho_{G_1}} - \frac{1}{\rho_{L_1}}$
5. $\rightarrow \Delta P_2 \quad P_2 = P_1 + \Delta P_2$

Choked if $\Delta P \rightarrow \infty$

Separated Pipe Flows

Each Phase Occupies a Specific Fraction of the Flow Area

"Two-Phase Multiplier" for friction loss:

$$\left(-\frac{\partial P}{\partial L}\right)_{fm} = \Phi_R^2 \left(-\frac{\partial P}{\partial L}\right)_{fR}$$

Reference Single-Phase Flow:

$R = L$ ***Total flow is liquid:*** $\left(G_L = \dot{m}_m / A\right) = G$

$R = G$ ***Total flow is gas:*** $\left(G_G = \dot{m}_m / A\right) = G$

$R = L_{Gm}$ ***Total flow is liquid in mixture:***

$$G_{Lm} = \left(1 - x\right)G$$

$R = G_{Gm}$ ***Total flow is gas in the mixture***

$$G_{Gm} = xG$$

Lockhart-Martinelli (1949)

$$\left(-\frac{\partial P}{\partial L}\right)_f = \Phi_{Lm}^2 \left(-\frac{\partial P}{\partial L}\right)_{fLm}$$

or:

$$\left(-\frac{\partial P}{\partial L}\right)_f = \Phi_{Gm}^2 \left(-\frac{\partial P}{\partial L}\right)_{fGm}$$

L-M Two-Phase Multipliers

$$\Phi_{Lm}^2 = 1 + \frac{C}{\chi} + \frac{1}{\chi^2}$$

$$\Phi_{Gm}^2 = 1 + C\chi + \chi^2$$

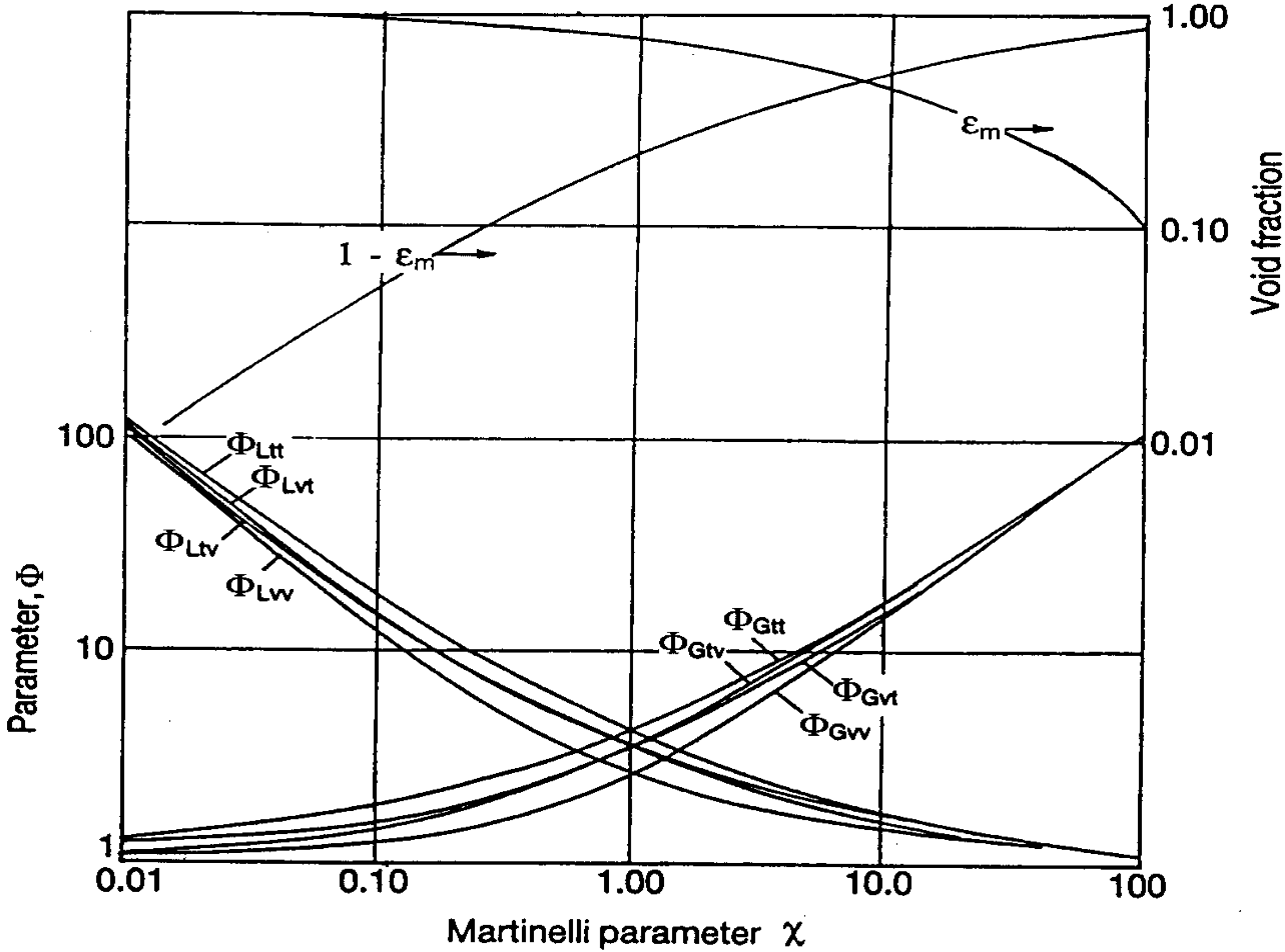
State	Liquid	Gas	C
tt	turbulent	turbulent	20
vt	laminar	turbulent	12
tv	turbulent	laminar	10
vv	laminar	laminar	5

L-M Correlating Parameter

$$\chi^2 \equiv \left(-\frac{\partial P}{\partial L}\right)_{f_{Lm}} \div \left(-\frac{\partial P}{\partial L}\right)_{f_{Gm}}$$

$$-\left(\frac{\partial P}{\partial L}\right)_{fLm} = \frac{2\, f_{Lm}\left(1-x\right)^2 G^2}{\rho_L D}$$

$$-\left(\frac{\partial P}{\partial L}\right)_{fGm} = \frac{2\, f_{Gm}\, x^2 G^2}{\rho_G D}$$



1.00
0.10
0.01
Void fraction
εm
1 - εm
100
10
1
Parameter, Φ
ΦLtt
ΦLvt
ΦLtv
ΦLvv
ΦGtt
ΦGtv
ΦGvt
ΦGvv
0.01
0.10
1.00
10.0
100
Martinelli parameter χ

Friction Factors

f_{Lm} is based on "liquid only" Reynolds No.

$$N_{Re_{Lm}} = \frac{(1-x)GD}{\mu_L}$$

f_{Gm} is based on "gas-only" Reynolds No.

$$N_{Re_{Gm}} = \frac{xGD}{m_G}$$

Duckler et al. (1964)

$$\Phi_{Lm}^{2}=\frac{\rho_{L}}{\rho}\alpha\left(\varphi\right)\beta\left(\varphi\right)$$

and

$$\Phi_{Gm}^{2}=\frac{\rho_{G}}{\rho}\alpha\left(\varphi\right)\beta\left(\varphi\right)$$

where

$\varphi=\left(1-\varepsilon\right),\ \rho$ ***are "no slip" values*** *and*

$$\alpha(\varphi) = 1 + \frac{-\ln\varphi}{1.281 - 0.478(-\ln\varphi) + 0.444(-\ln\varphi)^2 - 0.094(-\ln\varphi)^3 + 0.00843(-\ln\varphi)^4}$$

$$\beta(\varphi) = \frac{\rho_L}{\rho}\left(\frac{\varphi^2}{\varphi_m}\right) + \frac{\rho_G}{\rho}\left(\frac{(1-\varphi)^2}{1-\varphi_m}\right)$$

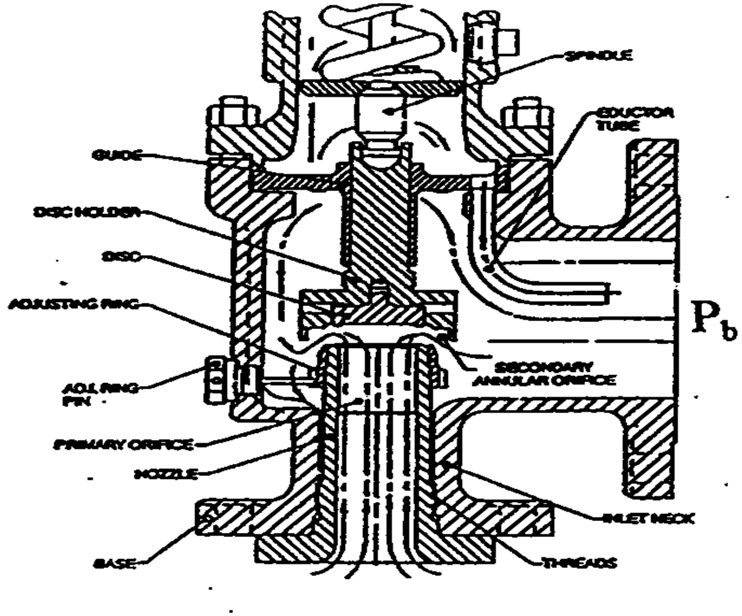
The Reynolds Number is based on mixture properties:

$$N_{Re_m} = \frac{DG}{\mu_m}\beta(\varphi)$$

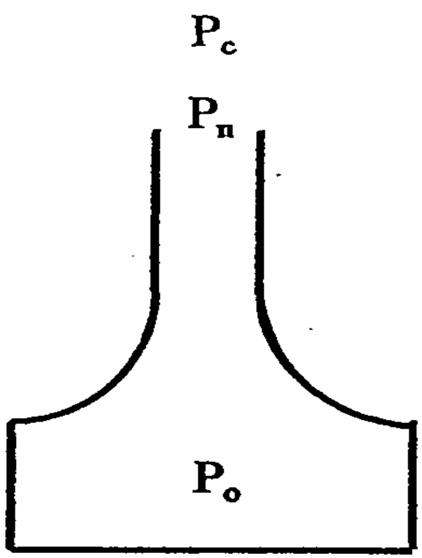
Sizing Relief Valves for Two-Phase Flow

$$A = \frac{\dot{m}}{G_{valve}}$$

$$G_{valve} = K_d G_{ideal\ nozzzle}$$



(a) Actual Safety Relief Valve



(b) Ideal Nozzle

Assume Homogeneous Gas-Liquid Mixture in an Isentropic Nozzle

$$G = K_d \rho_n \left(-2 \int_{P_o}^{P_n} \frac{dP}{\rho} \right)^{1/2}$$

Discharge Coefficient (K_d)

Values given by manufacturer, or in the "Red Book"

If flow is choked (critical) use :

$$K_{d,gas}$$

If flow is not choked (sub-critical) use :

$$K_{d,liquid}$$

TWO-PHASE DENSITY

$$\rho = \varepsilon\rho_G + (1-\varepsilon)\rho_L$$

Where ε ***is the volume fraction of gas:***

$$\varepsilon = \frac{x}{x + S(1-x)\rho_G / \rho_L}$$

x ***= mass fraction of gas phase (quality).***

S ***= slip ratio =*** v_G / v_L ***= (fn(x,*** ρ_L / ρ_G***, …etc)***

Flashing Flow

Non-Equilibrium If $L \leq 10$ cm

Flashing is not complete if $L \leq 10$ cm

In this case, use

$$x = x_o + \left(x_e - x_o\right)\frac{L}{10}$$

L **= nozzle length (cm)**

x_o **= initial quality entering nozzle**

x_e **= local quality assuming equilibrium**

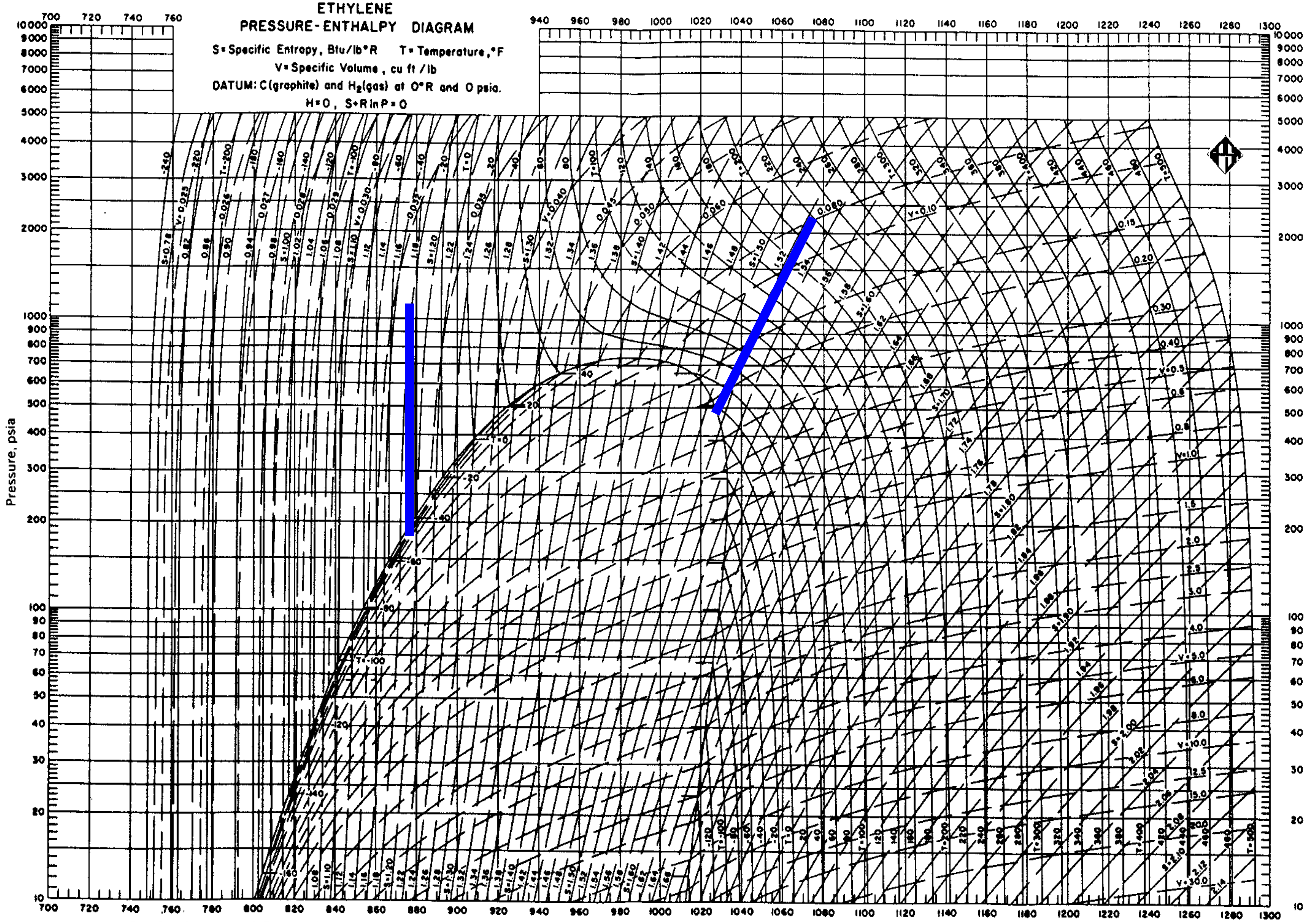
If $x_o > 0.05$, $x = x_e$

<u>Determine Quality</u>, $x_e = fn(P)$

- ***The quality is determined as a function of pressure by an energy balance on the fluid along the flow path.***

- ***The path is usually assumed to be isentropic.***

FIG. 17-24
ETHYLENE
PRESSURE-ENTHALPY DIAGRAM



Rev. 1981

From *Fluid Thermodynamic Properties for Light Petroleum Systems*, by Dr. K. E. Starling, copyright © Gulf Publishing Company; reproduced by permission.

HDI – Homogeneous Direct Integration

Exact Solution – Based on Numerical Finite Difference Equivalent of Nozzle Equation

$$G_n = \rho_n K_d \left(-2\int_{P_o}^{P_n} \frac{dP}{\rho} \right)^{1/2} = \rho_n K_d \left[-4\sum_{j=o}^{j=n-1} \left(\frac{P_{j+1} - P_j}{\rho_{j+1} + \rho_j} \right) \right]^{1/2}$$

Required Information:

ρ vs P at constant s from P_o to P_n in increments of P_j to P_{j+1}.

Can be generated from an EOS or from a database (e.g. steam tables).

(If choked, $G_n \rightarrow G_{max}$ at $P_n = P_c$)

Experimental Data

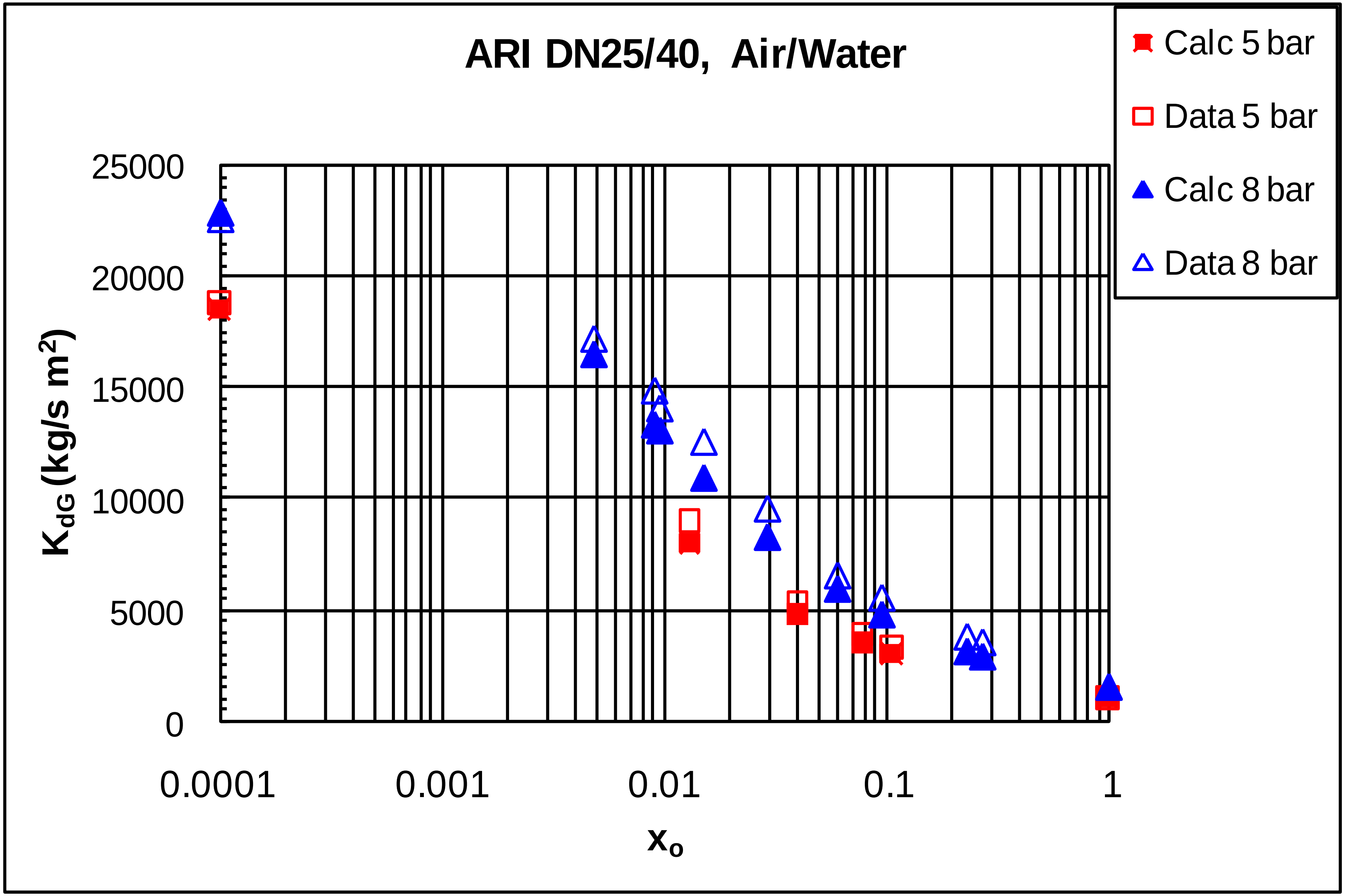
TABLE I VALVE SPECIFICATIONS (Lenzing, et al, 1997, 1998)				
Valve	K_{dG}	K_{dL}	Orifice Dia. (mm)	Orifice Area
B&R DN25/40 (Bopp & Reuther Si63)	0.86	0.66	20	0.4869
ARI DN25/40 (Albert Richter 901/902)	0.81	0.59	22.5	0.6163
1 x 2 "E" (Crosby JLT/JBS)	0.962	0.729	13.5	0.2219
Leser DN25/40 (441)	0.77	0.51	23	0.6440

TABLE II
FLOW CONDITIONS
(Lenzing, et al, 1997, 1998)

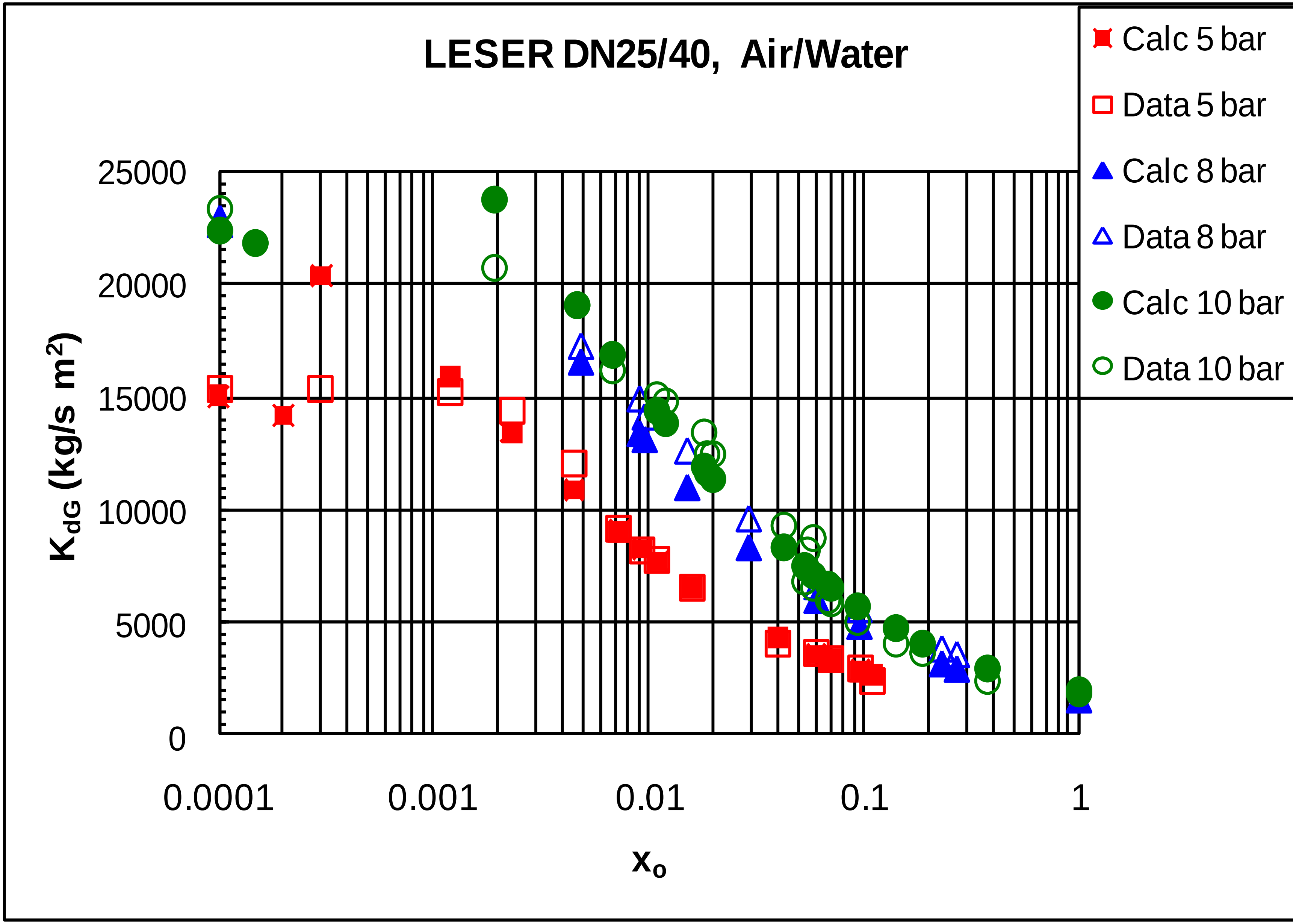
Fluid	Nom. Pressure (bar)	P_o (psia)	P_b (psia)
Air/Water	5	72.495	14.644
Air/Water	8	115.993	14.644
Air/Water	10	144.991	14.644
Steam/Water	5.4	78.295	14.644
Steam/Water	6.8	98.594	14.644
Steam/Water	8	115.993	14.644
Steam/Water	10.6	153.690	14.644

Air-Water (Frozen) Flow

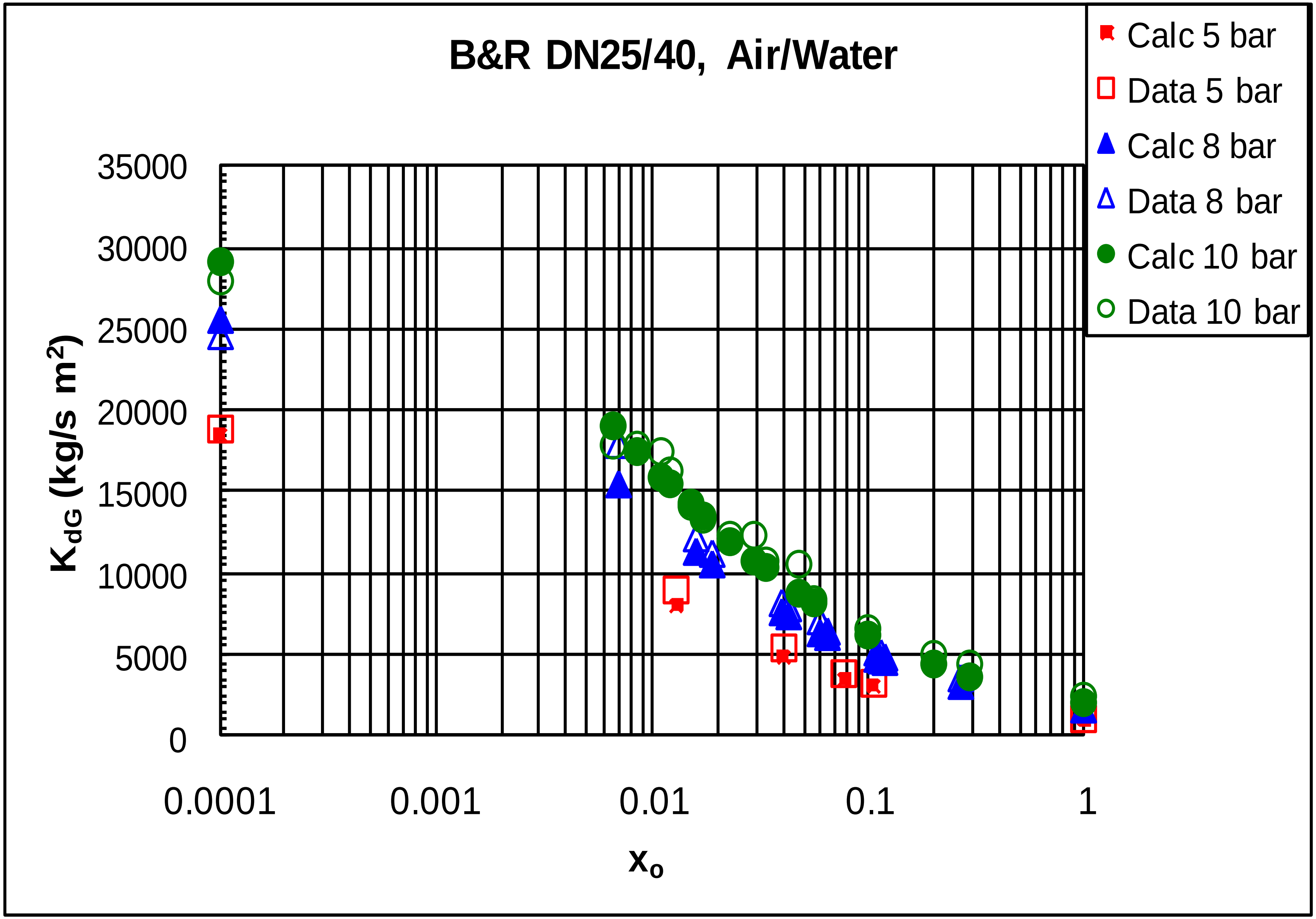
- ***Four Different Valves***
- ***Three Different Pressures***



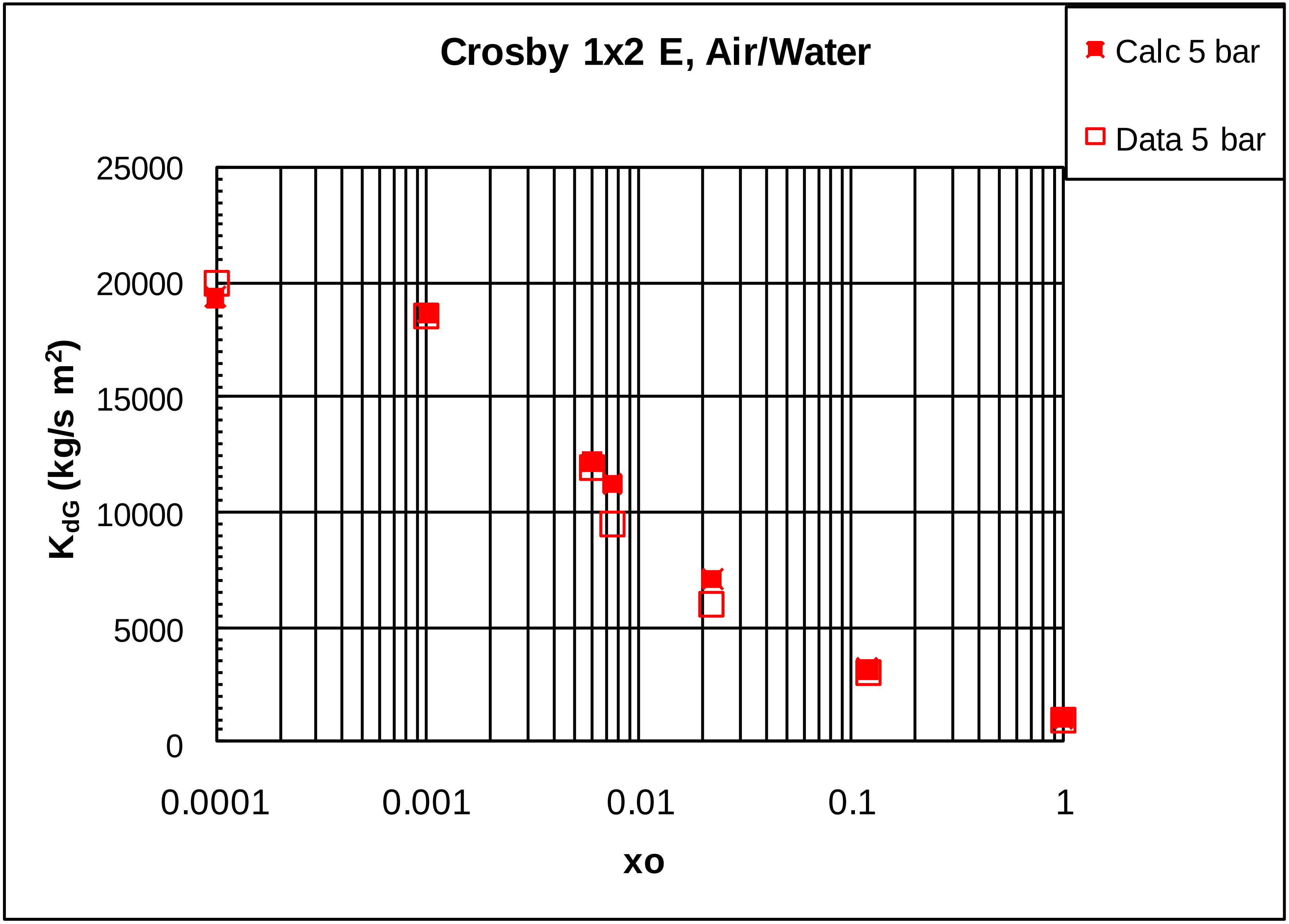
ARI DN25/40, Air/Water
Calc 5 bar
Data 5 bar
Calc 8 bar
Data 8 bar
25000
20000
15000
10000
5000
0
0.0001
0.001
0.01
0.1
1
K_dG (kg/s m²)
x_o



LESER DN25/40, Air/Water
Calc 5 bar
Data 5 bar
Calc 8 bar
Data 8 bar
Calc 10 bar
Data 10 bar
25000
20000
15000
10000
5000
0
0.0001
0.001
0.01
0.1
1
K_dG (kg/s m²)
x_o



B&R DN25/40, Air/Water
Calc 5 bar
Data 5 bar
Calc 8 bar
Data 8 bar
Calc 10 bar
Data 10 bar
35000
30000
25000
20000
15000
10000
5000
0
0.0001
0.001
0.01
0.1
1
K_{dG} (kg/s m^2)
x_o



Crosby 1x2 E, Air/Water
Calc 5 bar
Data 5 bar
K_{dG} (kg/s m^2)
25000
20000
15000
10000
5000
0
0.0001
0.001
0.01
0.1
1
xo

HNDI – Homogeneous Non-Equilibrium Direct Integration

*For **flashing flows**, equilibrium is not reached until flow path length reaches 10 cm or more.*

*For **L<10 cm**, quality (x = gas mass fraction) is lower than it would be at equilibrium (x_e).*

*For **L < 10 cm**, quality is estimated from*

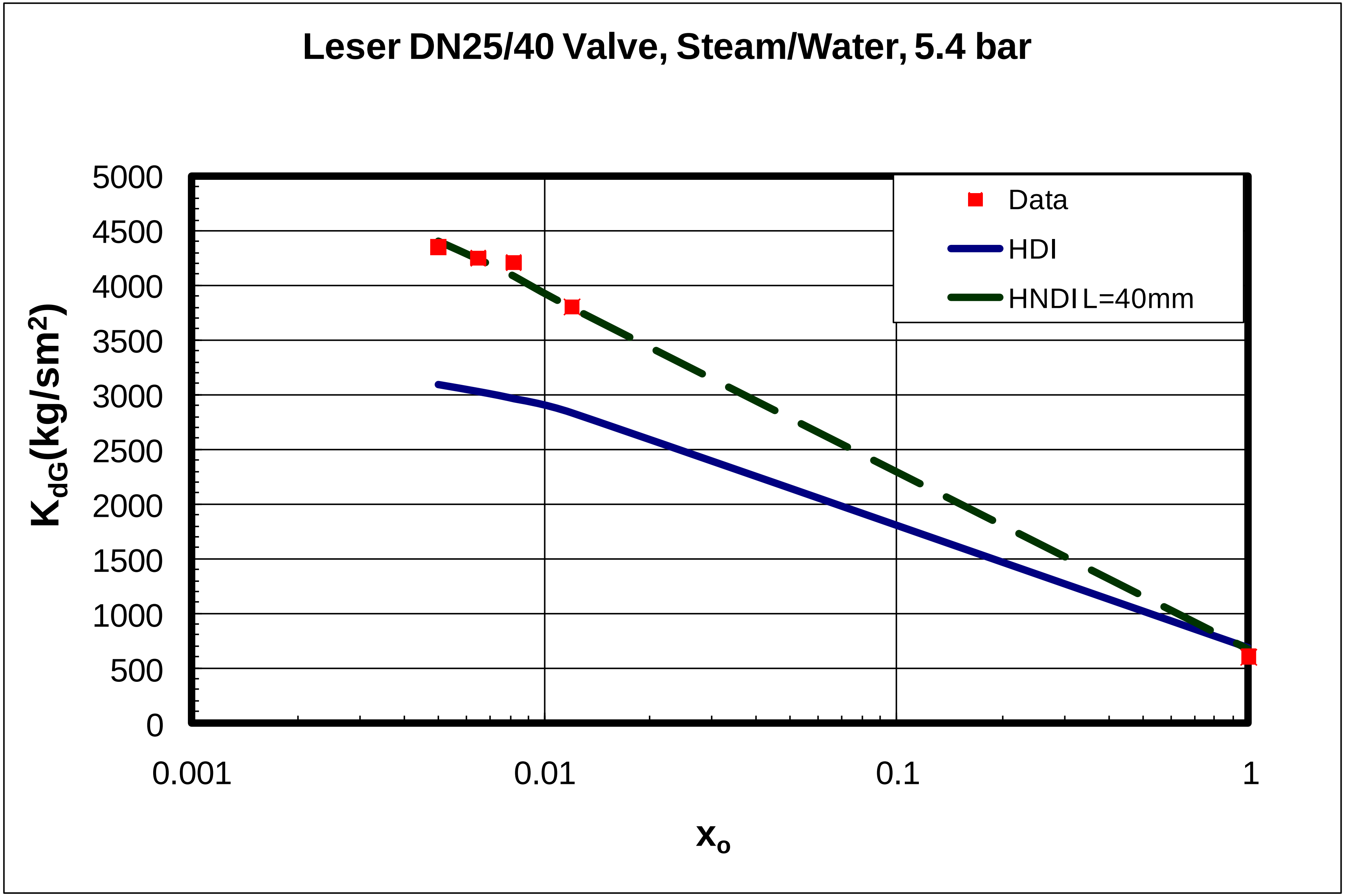
$$x = x_o + \left(x_e - x_o \right) L / 10$$

where x_o is the initial (L = 0) quality (L in cm)

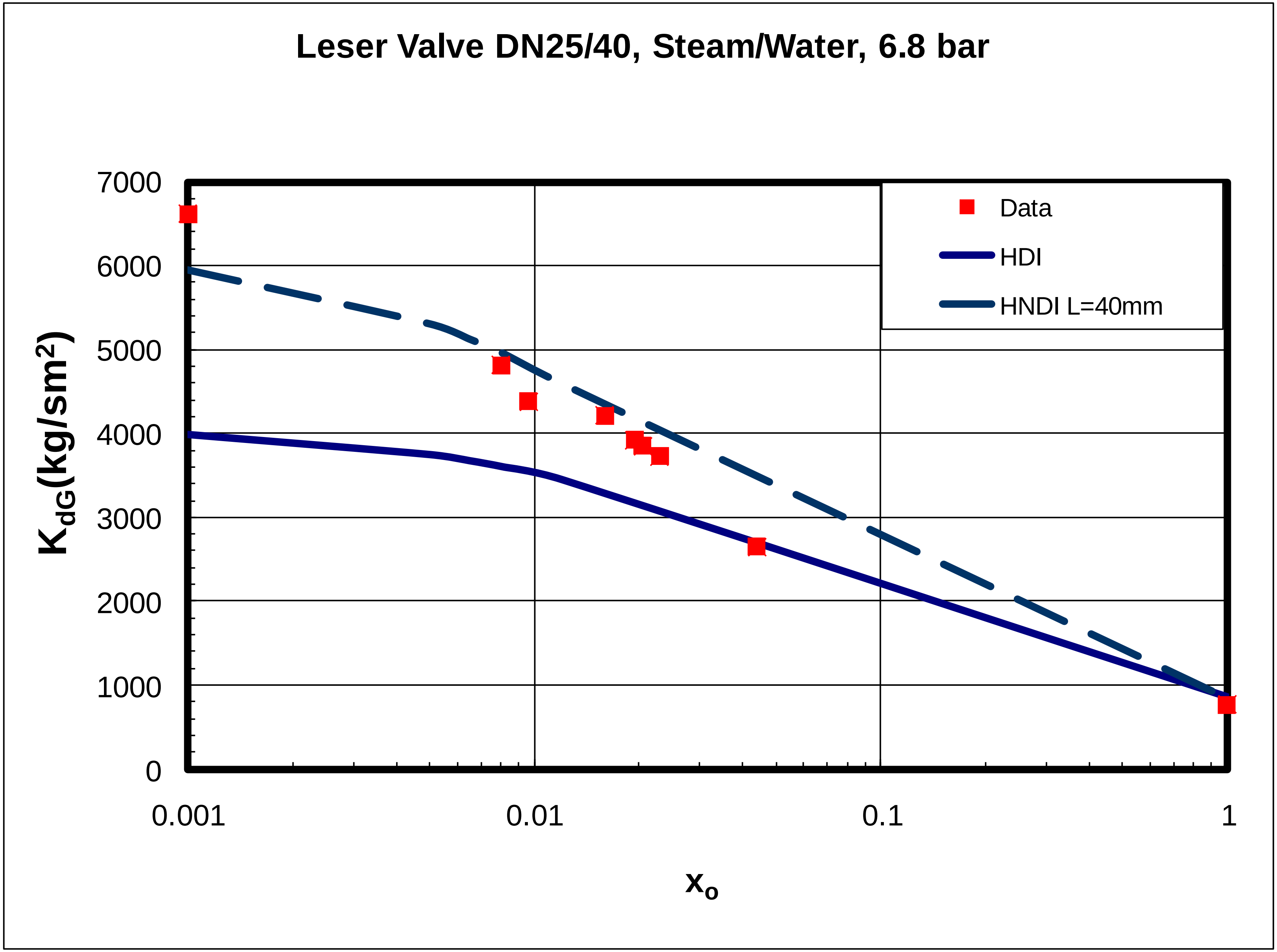
If $x_o > 0.05$, $x = x_e$

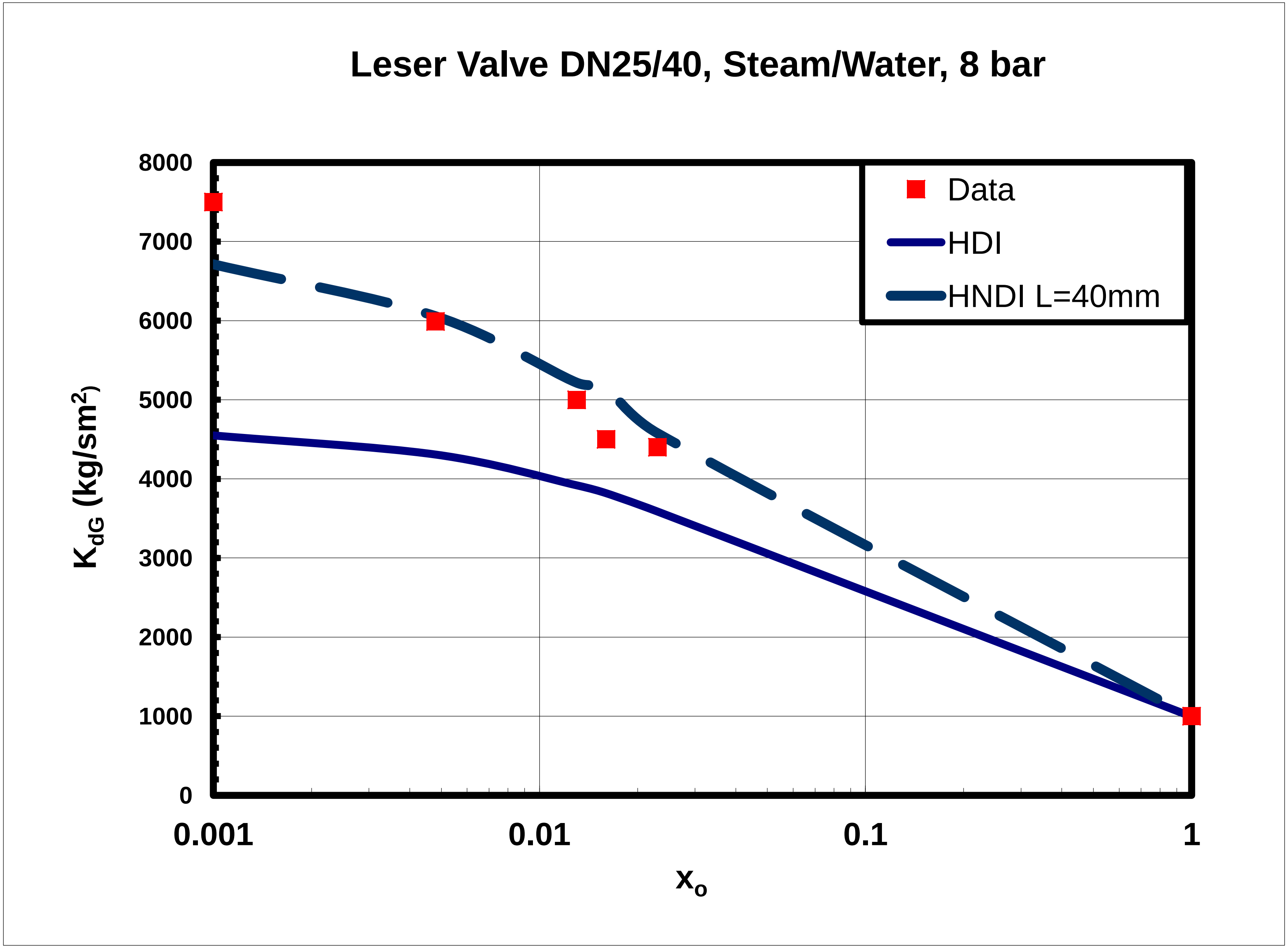
Steam-Water Flashing (non-Equilibrium) Flow

- ***One Valve - Leser 25/40***
- ***Four Different Pressur**es*

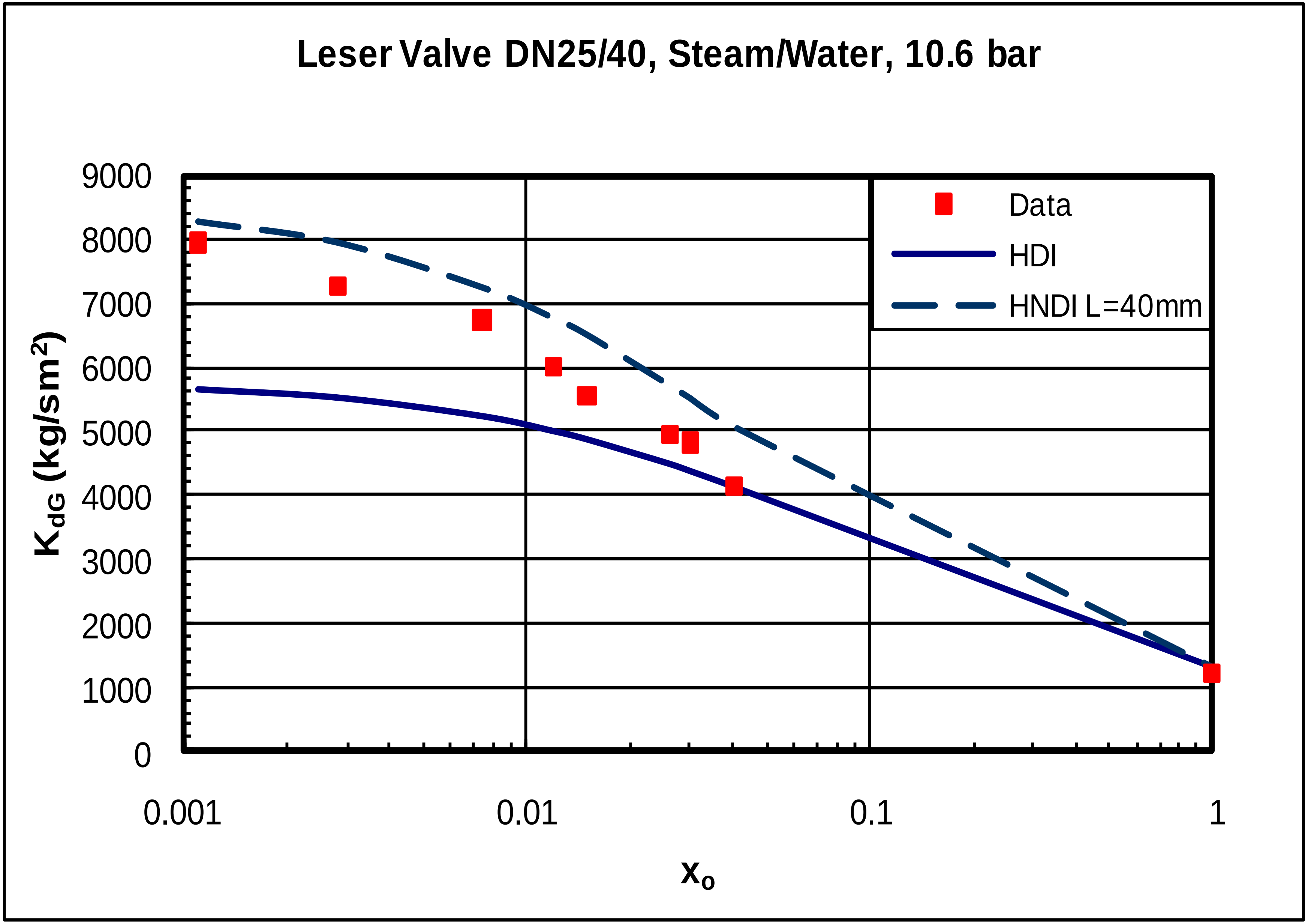


Leser DN25/40 Valve, Steam/Water, 5.4 bar
Data
HDI
HNDI L=40mm
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
K_{dG}(kg/sm^2)
0.001
0.01
0.1
1
x_o


Leser Valve DN25/40, Steam/Water, 6.8 bar
$K_{dG}(kg/sm^2)$
7000
6000
5000
4000
3000
2000
1000
0
0.001
0.01
0.1
1
x_o
Data
HDI
HNDI L=40mm



Leser Valve DN25/40, Steam/Water, 8 bar
8000
7000
6000
5000
4000
3000
2000
1000
0
0.001
0.01
0.1
1
K_{dG} (kg/sm^2)
x_o
Data
HDI
HNDI L=40mm



Leser Valve DN25/40, Steam/Water, 10.6 bar
Data
HDI
HNDI L=40mm
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
0.001
0.01
0.1
1
K_{dG} (kg/sm²)
x_o

SUMMARY/MORAL

- ***Two-Phase Flow is much more complex than single –phase flow, because of the wide variety of possible flow regimes, phase distributions, thermo/mechanical equilibrium/non-equilibrium, etc.***

- ***Correlations are complex and limited in scope.***

- ***Analysis requires good understanding of flow mechanism.***

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Questions??